METHOD OF CALCULATING SUPERSONIC GAS-DROPLET JETS

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A method is proposed for calculating two-phase supersonic jets with allowance for the effect of the liquid on the turbulence characteristics of the carrier phase.

A number of modern technical applications require a detailed description of the structure of supersonic jets bearing particles of atomized liquid fuel. It is known that the presence of a dispersed impurity in a turbulent flow in the form of solid particles or drops can have a substantial effect on the turbulence characteristics of the flow [1], which in turn determine the intensity of mixing processes. It was shown experimentally in [2] that supersonic gas-droplet flows are governed by the same laws as subsonic flows. An increase in the content of liquid in the jet leads to a deterioration in mixing, contraction of the jet, and an increase in its range. This indicates need for a further, more detailed accounting of the effect of phase interaction in both averaged and fluctuation motion.

The methods that have been developed for modeling two-phase flows can be divided into two broad categories [3]. In both cases, the parameters of the gas phase are calculated in an Eulerian formulation, and the conservation equations for the carrier phase differ from the standard Navier–Stokes equations only in the presence of additional source terms connected with the effects of phase interaction. Here, the discrete phase is examined either in the Eulerian formulation (continuum approximation) or in a Lagrangian formulation (trajectory method). An analysis of the literature shows that the Eulerian approach is more widely used, its convenience owing to the fact that the methods of integration that are employed are the same as for the carrier phase.

However, when calculations are being performed for two-phase flows in which the disperse phase has a polydisperse composition, it is necessary to integrate conservation equations for a large number of "classes" of particles – in accordance with their initial size distribution. This entails a significant increase in computing time, which is an obstacle to wider use of the given approach. Computing time can be reduced by one of two methods: change over to a Lagrangian formulation for the discrete phase with allowance for its instantaneous size distribution [3]; use more efficient algorithms for integration of equations written in the continuum approximation.

The Pantankar–Spalding method [4] has been widely used to successfully model one-phase subsonic and supersonic flows in the boundary-layer approximation. This method involves a transition to the coordinates (x, ω). Here, the conservation equations, written in generalized form in cylindrical coordinates

$$\rho U \frac{\partial \Phi}{\partial x} + \rho V \frac{\partial \Phi}{\partial y} = \frac{1}{y} \frac{\partial}{\partial y} \left(y \Gamma_s \frac{\partial \Phi}{\partial y} \right) + S_s, \tag{1}$$

appear as follows when written in the new coordinates:

$$\frac{\partial \Phi}{\partial x} + (a + b\omega) \frac{\partial \Phi}{\partial \omega} = \frac{\partial}{\partial \omega} \left(c \frac{\partial \Phi}{\partial \omega} \right) + d. \tag{2}$$

Equation (2) does not contain the transverse velocity of the gas and can be solved by the Crank–Nicholson method with second-order accuracy. Here, no iterations are performed and computing time is reduced appreciably.

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For two-phase flows, this method can be used without modification only within the framework of the "passive impurity" model – when the velocities of the phases are equal [5]. In the presence of phase-velocity nonequilibrium, changing from Eq. (1) to Eq. (2) will not eliminate the transverse component in the conservation equations for the disperse phase and thus allow use of the advantages of the method. This problem can be circumvented if we solve the equations of the disperse phase on a separate finite-difference grid in the coordinate system (x, ω_p). Here, by analogy with the gas phase

$$\omega_p = \frac{\psi_p - \psi_{pi}}{\psi_{pe} - \psi_{pi}}. \tag{3}$$

In the above equation, φ_p is the stream function for the disperse phase. It is determined as

$$\psi_p = \int\limits_{r_i}^{r_e} \rho_p U_p dy. \tag{4}$$

The indices e and i correspond to the external and internal boundaries of the mixing layer.

In the solution of the equations for the dispersion medium and disperse phase, the exchange terms in (2) are determined on separate finite-difference grids by mutual interpolation of the independent variables at each integration step after calculation of the radial coordinates of the nodes on these grids (the longitudinal coordinates of the nodes coincide).

The procedure just described can be performed for each particle if the polydisperse composition of the discrete phase is broken down into several "classes." In the present study, we will attempt to simplify our analysis of the possibilities of the given method by assuming that the droplets of the liquid phase are monodisperse. We also made the following assumptions: 1) the boundary-layer approximation can be used; 2) the disperse phase is continuous and is characterized by distributed density ρ_p, mean droplet velocity U_p, and mean droplet temperature T_p; 3) coalescence and fragmentation of the droplets can be ignored; 4) slip of the particles is considered only in the longitudinal direction; 5) the dynamic interaction of the phases is determined by the resistance force; 6) the droplets are considered to be spherical, with a size equal to the mean volume-surface diameter.

Written in cylindrical coordinates, the system of conservation equations for the gas phase includes the continuity equation

$$\frac{\partial}{\partial x}(\rho U) + \frac{1}{y}\frac{\partial}{\partial y}(y\rho V) = 0, \tag{5}$$

the projection of the momentum equation on the x axis

$$\rho U \frac{\partial U}{\partial x} + \rho V \frac{\partial U}{\partial y} = \frac{1}{y}\frac{\partial}{\partial y}\left(y\mu_{\text{ef}} \frac{\partial U}{\partial y} \right) - F_x. \tag{6}$$

and the energy equation

$$\begin{gathered} \rho U \frac{\partial H}{\partial x} + \rho V \frac{\partial H}{\partial y} = \frac{1}{y}\frac{\partial}{\partial y}\left(y \frac{\mu_{\text{ef}}}{\text{Pr}_{\text{ef}}} \frac{\partial H}{\partial y} \right) + \\ + \frac{1}{y}\frac{\partial}{\partial y}\left\{ \mu_{\text{ef}} \left[\left(1 - \frac{1}{\text{Pr}_t} \right) U \frac{\partial U}{\partial y} + \left(\frac{1}{\sigma_k} - \frac{1}{\text{Pr}_t} \right) \frac{\partial K}{\partial y} \right] \right\} + F_x(U - U_p). \end{gathered} \tag{7}$$

This system of equations is supplemented by the equation of state for the gas and the well-known Lounder–Jones two-parameter turbulence model. Here, the model has additional terms connected with the presence of a second phase:

$$\rho U \frac{\partial K}{\partial x} + \rho V \frac{\partial K}{\partial y} = \frac{1}{y}\frac{\partial}{\partial y}\left(y \frac{\mu_{\text{ef}}}{\text{Pr}_{\text{ef}}} \frac{\partial K}{\partial y} \right) + \mu_t \left(\frac{\partial U}{\partial y} \right)^2 - \rho(\varepsilon + \varepsilon_p), \tag{8}$$

$$\rho U \frac{\partial \varepsilon}{\partial x} + \rho V \frac{\partial \varepsilon}{\partial y} = \frac{1}{y} \frac{\partial}{\partial y}\left(y \frac{\mu_{\text{ef}}}{\text{Pr}_{\text{ef}}} \frac{\partial \varepsilon}{\partial y} \right) + C_{\varepsilon 1} \left(\frac{\varepsilon}{K} \right) \mu_t \left(\frac{\partial U}{\partial y} \right)^2 - \tag{9}$$

$$- C_{\varepsilon 2} \left(\frac{\varepsilon}{K} \right) \rho \varepsilon - \rho F_p,$$

$$\mu_t = \rho C_\mu K^2 / \varepsilon, \ \ \mu_{\text{ef}} = \mu_t + \mu_l. \tag{10}$$

We used the usual group of constants in the turbulence model. The additional terms ε_p and F_p were calculated in accordance with [6]. The correction for supersonic compressibility was accounted for in the form of the dependence of the coefficient C_μ on the Mach number of the flow [7].

The system of averaged equations for the disperse phase includes the following:

the continuity equation

$$\frac{\partial}{\partial x}(\rho_p U_p) + \frac{1}{y} \frac{\partial}{\partial y}(y \rho_p V^*) = 0, \tag{11}$$

where

$$V^* = V_p + \overline{\rho_p' v_p'} / \rho_p, \ \ V_p = V,$$

the momentum equation projected on the x axis

$$\rho_p U_p \frac{\partial U_p}{\partial x} + \rho_p V^* \frac{\partial U_p}{\partial y} = \frac{1}{y} \frac{\partial}{\partial y} (- y \rho_p \overline{U_p' V_p'}) + F_x, \tag{12}$$

the energy equation

$$\rho_p U_p \frac{\partial T_p}{\partial x} + \rho_p V^* \frac{\partial T_p}{\partial y} = \frac{1}{y} \frac{\partial}{\partial y} (- y \rho_p \overline{T_p' V_p'}). \tag{13}$$

To realize the proposed method, Eqs. (11)-(13) must be reduced to a form analogous to (1).

We now introduce the ratio of the density of the gas to the distributed density of the droplets: $\varkappa = \rho/\rho_p$. Then using a gradient representation for the correlation $\overline{\rho_p' V_p'} = -D_p \partial \rho_p / \partial y$, we find from Eqs. (5), (11) that

$$\rho_p U_p \frac{\partial \varkappa}{\partial x} + \rho_p V^* \frac{\partial \varkappa}{\partial y} = \frac{1}{y} \frac{\partial}{\partial y} \left(y D_p \rho_p \frac{\partial \varkappa}{\partial y} \right) - \frac{1}{y} \frac{\partial}{\partial y} \left(y D_p \frac{\partial \rho}{\partial y} \right) +$$

$$+ \frac{\partial}{\partial x} [\rho (U_p - U)]. \tag{14}$$

The correlation $\overline{U_p' V_p'}$ in Eq. (12) is proportional to the corresponding correlation for the gas. The diffusion coefficient of the particles D_p and the proportionality factor A (which is a function of the aerodynamic relaxation time of the particles and the Reynolds number in relative motion) are calculated as they were in [6]. Here, Eq. (12) can be written in the form (1), where the diffusion term is determined as

$$\Gamma_{up} = \frac{\rho_p}{\rho} A \mu_t \left(\frac{\partial U}{\partial y} \right) \Big/ \left(\frac{\partial U_p}{\partial y} \right). \tag{15}$$

The ratio of the gradients of the velocities of the disperse phase and dispersion medium can be determined from the previous integration step. Equation (13) need not be examined for the isothermal case.

The proposed flow model was first of all substantiated for the case of the discharge of a submerged supersonic one-phase jet with M = 1.5. Figure 1 (curve 1) compares the decay of the axial velocity of the gas in a supersonic jet with an initial velocity of 420 m/sec and experimental data obtained on the unit described in [2]. The satisfactory agreement between

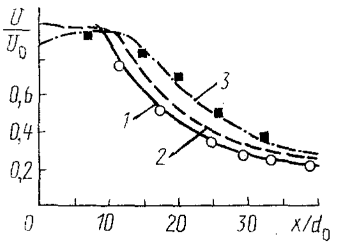

Fig. 1

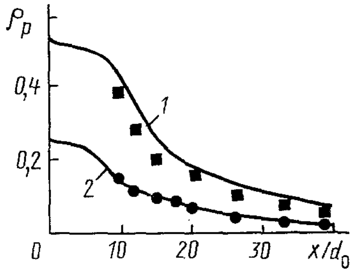

Fig. 2

Fig. 1. Axial profiles of velocity in submerged supersonic gas (1) and gas-droplet (2 and 3) jets.

Fig. 2. Change in the distributed density of the liquid phase along the jet axis: 1) $G_p = 17.7$ g/sec; 2) 7.9. ρ_p, kg/m^3.

the theoretical and experimental (clear points) data confirms the correctness of the choice of turbulence model and the correction for supersonic compressibility.

The same figure shows profiles of gas velocity (2) and droplet velocity (3) in a gas-droplet jet in which the initial rate of flow of kerosine TS-1 was $G_p = 0.0079$ kg/sec (air flow rate $G = 0.073$ kg/sec). The presence of the drops of liquid increases the range of the jet, which is consistent with existing representations on the suppression of turbulence by particles of a second phase. The dark points show the mean velocity of the drops as measured by the time-of-flight method [8].

Figure 2 shows the change of the distributed density of kerosine along the jet axis for two values of initial liquid flow rate. Also shown is experimental data obtained by the method of low-angle scattering and attenuation of laser radiation [2]. The qualitative and quantitative agreement between the theoretical and empirical values of mean velocity and the distributed density of the drops indicates the adequacy of the chosen model of the aerodynamic interaction of the phases, turbulent diffusion of the drops, and the effect of the liquid phase on the turbulence structure of the jet.

The time of calculation of a typical variant by the proposed method on an IBM PC AT is no greater than 3 min, which makes it possible to recommend the method for more complex classes of flows – such as the vaporization and combustion of liquid fuels with a polydisperse drop size in a turbulent flow.

NOTATION

a, b, c, d, coefficients in the generalized equation; $C_{\varepsilon 1}$, $C_{\varepsilon 2}$, $C\mu$, σ_k, constants in the turbulence model; D_p, diffusion coefficient of the particles; F_x, phase interaction force; G_p, flow rate of the liquid phase; H, total enthalpy of the dispersion medium; K, kinetic turbulence energy; Pr, Prandtl number; S_s, source term in the generalized equation; T, temperature; U, V, longitudinal and transverse components of velocity; x, y, longitudinal and transverse coordinates; ε_p, F_p, additional terms in the turbulence model connected with the presence of the second phase; ε, rate of dissipation of turbulence energy; μ, viscosity coefficient of the gas; ψ, stream function; $\varkappa$, inverse of the mass concentration of droplets; ρ, density; ω, dimensionless stream function; Γ_s, diffusion term in the generalized equation; Φ, generalized variable. Indices: p, pertaining to the disperse phase; ef, effective value; t, turbulent value.

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